Derivada de un vector respecto a un escalar: $\lim\limits_{\Delta t \to 0} \frac{\Delta \boldsymbol{v}}{\Delta t} = \lim\limits_{\Delta t \to 0} \frac{\boldsymbol{v}(t+\Delta t) - \boldsymbol{v}(t)}{\Delta t} = \frac{d\boldsymbol{v}}{dt}$

las componentes coordenadas de $d\boldsymbol{v}/dt$ serán: $\frac{dv_x}{dt} \qquad \frac{dv_y}{dt} \qquad \frac{dv_z}{dt}$

Propiedades:

a) Si $\boldsymbol{v} = \boldsymbol{v}(s)$ y $s = s(t)$ obtenemos que: $\frac{d\boldsymbol{v}}{dt} = \frac{d\boldsymbol{v}}{ds}\frac{ds}{dt}$

b) Si $\boldsymbol{v}(t) = \boldsymbol{a}(t) + \boldsymbol{b}(t)$ tendremos: $\frac{d\boldsymbol{v}}{dt} = \frac{d\boldsymbol{a}}{dt} + \frac{d\boldsymbol{b}}{dt}$

c) Si $\boldsymbol{a}(t) = f(t)\,\boldsymbol{b}(t)$ tendremos: $\frac{d\boldsymbol{a}}{dt} = \frac{df}{dt}\boldsymbol{b} + f\frac{d\boldsymbol{b}}{dt}$

d) Derivada del producto escalar: $\frac{d(\boldsymbol{a}\cdot\boldsymbol{b})}{dt} = \boldsymbol{a}\cdot\frac{d\boldsymbol{b}}{dt} + \frac{d\boldsymbol{a}}{dt}\cdot\boldsymbol{b}$

e) Derivada del producto vector: $\frac{d(\boldsymbol{a}\times\boldsymbol{b})}{dt} = \boldsymbol{a}\times\frac{d\boldsymbol{b}}{dt} + \frac{d\boldsymbol{a}}{dt}\times\boldsymbol{b}$

Consecuencia: La condición que cumple un vector de dirección constante es también que: $\boldsymbol{v}\times\frac{d\boldsymbol{v}}{dt} = \boldsymbol{0}$

En efecto: si $\boldsymbol{v} = v\boldsymbol{u}$ tendremos que: $\frac{d\boldsymbol{v}}{dt} = \frac{dv}{dt}\boldsymbol{u} \quad \Rightarrow \quad \boldsymbol{v}\times\frac{d\boldsymbol{v}}{dt} = v\boldsymbol{u}\times\frac{dv}{dt}\boldsymbol{u} = v\frac{dv}{dt}(\boldsymbol{u}\times\boldsymbol{u}) = \boldsymbol{0}$ c.q.d.

Integración: $\boldsymbol{I} = \int_a^b \boldsymbol{v}(t)\,dt = \lim\limits_{\Delta t\to 0}\sum_i \boldsymbol{v}(t)\,\Delta t$

en función de sus componentes coordenadas: $\boldsymbol{I} = \boldsymbol{I}_x + \boldsymbol{I}_y + \boldsymbol{I}_z = \int_a^b \boldsymbol{v}_x(t)\,dt + \int_a^b \boldsymbol{v}_y(t)\,dt + \int_a^b \boldsymbol{v}_z(t)\,dt$